

Determination of Non-Linearity in Rail Pad

S.S. Sarawade¹, S.K. Agarwal², P. N. Gawarshettiwar¹, S. A. Patil¹, A. R. Yeole¹,
S.A.Jain¹

¹(Mechanical Engineering Department, M.E.S. College of Engineering, Pune, Maharashtra, India)

²(Bridge Engineering Department, IRICEN, Pune, Maharashtra, India)

Abstract: The grooved rail pad is made of rubber which is non-linear visco-elastic material. Toe load will depend upon total thickness of liner, rail foot and rubber pad. In present study, non-linear behavior of rail pad is simulated and toe load vs. variation in thickness is calculated. For above study, special fixture is developed and test is conducted on rail pad in UTM. Similarly, the load vs. deformation of fixture was simulated in ANSYS for same loading step and hence the actual load vs. deflection of rail pad is obtained with UTM machine, according exponential model of rubber which will be utilized for further calculations. CATIA and ANSYS are used for developing model.

Keywords: Rail fastening, Non-linear, Simulation, ANSYS, FEM

I. Introduction

In Indian railway, rail is attached to sleeper with elastic rail clip, a grooved rubber pad is provided in between rail and sleeper. The arrangement of elastic rail clip and rubber pad is such that standard toe load is available all the time in order to avoid indent and graining mark on rail. A liner is provided in between elastic rail clip and rail foot. The railway track is usually assumed to be linear in order to simplify the track model but rail pad and pandrol clip, ballast structure are actually behave non-linearly[1], this may cause incorrect result in some circumstances as pad and clip stiffness varies with load. Thus, to obtain the correct behavior of rail-pad, non-linear analysis of rail-pad is performed. Rail pad is made of visco-elastic material [1]. Toe load will depends upon the non-linear stiffness of material. Therefore an iterative approach is to be adopted to find out the actual toe load due to variation in the total thickness. The compression of rail-pad depends upon the toe load.

In the present study, non-linear behavior of elastic rail pad is simulated and toe load vs. variation in thickness is calculated. For above study, specially developed fixture plate having circular cross section is used during compression test on rail pad. Similarly the load vs. deformation of fixture is simulated in ANSYS and load vs. deflection variation obtained for same loading steps. Track deflection under an applied vertical load is necessary to assess the structural conditions of the track. Thus, non-linear elastic foundation is considered to be an important parameter to analyze and study the influence of non-linear stiffness of rail pad for deflection of rail structure [2, 10]. Analysis of track model is therefore needed to take account of non-linear properties of rail pad and their effects on deflection of rail along the length [3]. Rail-pad stiffness represents the proportion between vertical load and deflection. Winkler's approach assumes that the elastic foundation is a system of identical, independent, closely spaced and linear stiffness elastic rail structures. While modern approach of stiffness definition includes inelastic and non-linear behavior of track elements [4, 6]. Value of beam on elastic foundation thus obtained from analysis of rail with non-linear rail-pad support, determines the length of elastic support over which load is distributed [6]. The results obtained will vary for non-linear support as compare to linear one. The characteristics of load-displacement relationship of rail pad are determined from the compression test. The exponential equation for stiffness-displacement behavior is derived with necessary parameters obtained by curve fitting of experimental data. The stress-strain response of hyper-elastic and viscoelastic materials under uniaxial compression can be described by number of hyper-elastic models [7, 8]. In this paper material modeling of polynomial hyper elastic model is carried out to validate the results.

II. Hyperelasticity Of Rail Pad

For the ease of calculations, rail pad properties are widely assumed as linear, but actually it behaves hyper-elastic non-linear in nature. Here, a linear elastic model fails to describe accurately the real material behavior, whose stress-strain relationship is independent of strain rate. There are various material models available to determine above mentioned characteristics. Some of the models are:

- a) Saint-Venant-Kirchhoff model
- b) Neo-Hookean model
- c) Mooney Rivlin model
- d) Ogden material model

e) Polynomial model

Due to non-linear behavior stiffness of rail pad varies non-linearly and it has different value at every single point on the load vs. deflection curve. Practically it is difficult to find out the stiffness value for each and every loading step of curve. Hence to simplify this problem non-linear material modeling needs to perform.

2.1. Material modeling:

For the current study, we are following polynomial 3rd order approach to find out the variable stiffness of rail-pad.

Mathematical form of strain-energy potential for the polynomial option is given as.

$$W = \sum_{i+j=1}^N C_{ij} (I1 - 3)^i \cdot (I2 - 3)^j + \sum_{k=1}^N \frac{1}{dk} (J - 1)^{2k} \dots [8]$$

Where, W= strain energy potential.

I1= First deviatric strain invariant

I2=second deviatric strain invariant

J= Determinant of the elastic deformation gradient F

N, Cij= material constants

In general number of constants depends on value of N.

N= 3, C10,C01,C20,C02,C30,C21,C12,C03,d2,d3.

... [Table1]

The load vs. deflection nature obtained from the uniaxial loading test is considered as input to ANSYS software to find out above constants by curve fitting method. This modeling is carried out to validate the Load vs. deflection curve obtained by ANSYS with experimental one.

III. Beam On Elastic Foundation (BOEF)



Fig.1 Beam on Elastic Foundation [10]

In present study, rail is considered as a beam of infinite length simply placed on track structure of foundation. Winkler in 1867 stated that the beam lies on elastic foundation under the application of external loading, the reaction of the foundation are proportional at every point to deflection of beam at same point.

$$\beta = \sqrt[4]{\frac{K}{4EI}} \dots [10]$$

For rail, E = Modulus of Elasticity,

I = Moment of Inertia,

K = Spring Stiffness

β value of above equation indicates the length of beam on elastic foundation. In BOEF theory, straight beam supported by number of elastic medium (spring) is considered and load is applied over the beam. As the external loadings acts over the beam, it will deflect continuously distributed reaction forces in the spring support. Downward deflection indicates that there will be compression while it is assumed that tension produced due to upward deflection is not possible. Thus, the length calculated from the equation is assumed to be elastic medium and able to take up forces.

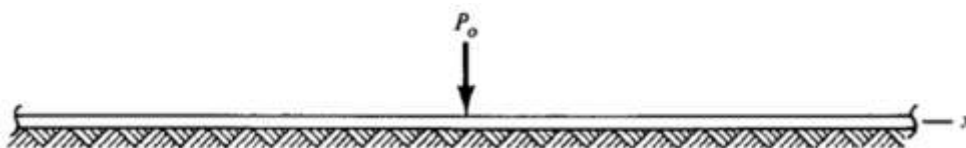


Fig.2 Schematic of Long Beam on Elastic Foundation subjected to Concentrated Load at Mid span [10]

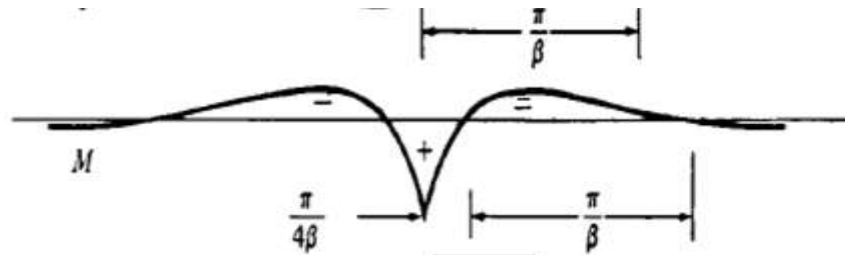


Fig.3 Displacement Variation along the length of Rail [10]

40m long beam having uniform cross section supported by spring supports at a distance of 0.65 between two successive springs is simulated in ANSYS. 250KN of load is applied at the center of beam thus indicates the load vs. deflection behavior along the beam.

Linear value of BOEF is then derived using the standard equation.

$$y = \frac{\beta \cdot p}{2K} \quad \dots [10]$$

p= Load applied,
y= Maximum Displacement,
K= Spring Stiffness

It is theoretical approach towards finding the maximum deflection, point of zero deflection in the structure.

Theoretical Calculations:

Linear Rail-pad:

$$E = 2 \times 10^5 \text{ N/mm}^2 = 2 \times 10^{11} \text{ N/m}^2$$

$$I = 3.047 \times 10^7 \text{ mm}^4 = 3.047 \times 10^{-5} \text{ m}^4$$

$$k = 60000 \text{ N/mm} = 60 \times 10^6 \text{ N/m}$$

$$\beta = \sqrt[4]{\frac{K}{4EI}}$$

$$\beta = \sqrt[4]{\frac{60 \times 10^6}{4 \times 2 \times 10^{11} \times 3.047 \times 10^{-5}}} = 1.2525 \text{ m}$$

$$y = \frac{\beta \cdot p}{2K}$$

y1= 0.1043 mm	y6= 0.6262 mm
y2= 0.2087 mm	y7= 0.7306 mm
y3= 0.3131 mm	y8= 0.835 mm
y4= 0.4175 mm	y9= 0.9393 mm
y5= 0.5218 mm	y10= 1.043 mm

IV. Experimental Study

The track model is consisting of a beam on a spring foundation representing the rail pad. Static stiffness increases linearly with load but due to non-linear properties of visco-elastic material (elastomers), dynamic stiffness at higher frequencies or higher load steps is generally higher than the static stiffness. Non-linear properties or dynamic stiffness of rail pad can be determined by performing material modeling. Dynamic stiffness is measured by conducting compression test over rail pad using UTM. Behavior of stiffness (Kp) and compression (Xp) is then observed and plot in mat lab. From mat lab, it has been observed that stiffness of rail pad increases exponentially with load as represented in equation. Tangent stiffness is given by (Kp= dp/dx), the stiffness-deflection equation can be derived as,

$$Kp = e^{b \cdot xp} \quad \dots [1]$$

Equation mentioned above represents an expression for the dynamic stiffness of rail pad using curve fitting procedures in mat lab. As previously discussed, the non-linear behavior of rail-pad tends to error in calculations. Hence in account to obtain the non-linear characteristics of the rail pad material experimental approach is adopted. To obtain load vs. deflection curve test is conducted on universal testing machine with arrangement of fixtures.

Aims of testing:

- To obtain stiffness values for rail-pad in order to compare the test values to those obtained by ANSYS simulation.
- To determine non-linear behaviour of rail-pad

For testing of rubber pad, it simply considered the pad upper surface area without grooves on it.

$$\begin{aligned} \text{S/F Area} &= l*b = 17.5 * 12.5 = 218.75 \text{ cm}^2 \\ &= 21875 \text{ mm}^2 \end{aligned}$$

Hence for the distribution of uniform pressure on the rail pad and to accommodate the deformation of rail-pad, 2 circular plates of mild steel having area 4 times more than rail pad are selected.

No. of plates = 2

t = thickness = 12 mm

Area of plate = 4*21875 = 87500 mm²

d = 350 mm

$$A = \frac{\pi}{4} d^2 = 87500 \text{ mm}^2$$

Test on rail pad carried out under normal environment condition of temperature = 298 K and atmospheric pressure.

Nature of loading = compressive

Magnitude of Load = 25 tonne = 250 KN

4.1 Test Results:

Test is carried out by placing rail pad in between two fixture plate. A load of 250 KN with graduated stepping applied on it and plot of load vs. deflection is obtained. Test is conducted under the supervision of lab in charge.

Test graph:

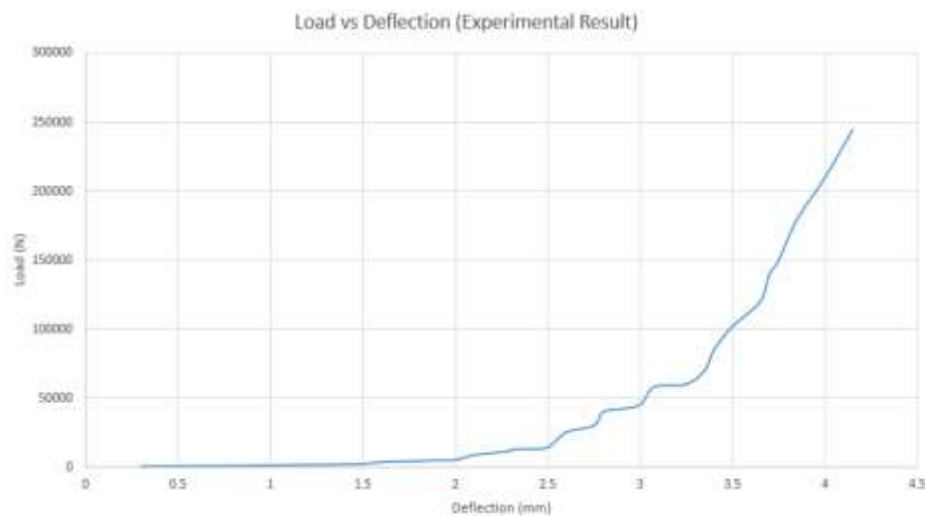


Fig.4 Load vs. Deflection Experimental Result

Derivation of Stiffness Equation in Mat Lab:

Stiffness of rail-pad is usually assumed to increase linearly with load to simplify analysis of track structure. Actual behavior of rail pad can be determined by derivation of rail pad stiffness from the test results. The tangent stiffness is given by $K_p = dp/dx$. Load deflection equation is determined by performing curve fitting procedure in Mat lab software.

Load deflection and Stiffness deflection equations can be derived as

Power equation ($y = a \cdot e^{bx}$) is $y = 0.256580 \cdot e^{1.694267 \cdot x}$

$$K_p = \frac{dy}{dx} = 0.4337 \cdot e^{(1.694267 \cdot x)}$$

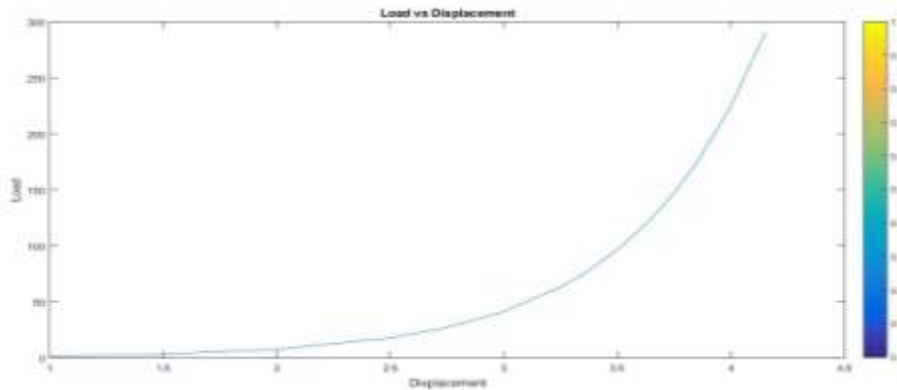


Fig.5 Load vs. Displacement graph using curve-fitting procedure in mat-Lab

V. Simulation Using Software

The same test is then simulated using ANSYS software so as to verify the results comparing with experimental. Load vs. deflection data from test is given in tabular form as material characteristics to find out best curve fit values. Graph obtained from experimental study gives values of load vs. deflection to ANSYS material library as input and solved curve fit for polynomial 3rd degree equation. Values of all the constants have been obtained from the solutions. Hence, the material properties for rail pad are considered by software. By considering the entire test parameters load vs. deflection plot obtained.

Table 1: Representation of Material constants

MATERIAL CONSTANTS	MPa	MATERIAL CONSTANTS	MPa
C10	-25.48	C12	21008
C01	26.42	C03	-6278.5
C20	-1.27e5	D1	0
C02	2.54e5	D2	0
C30	-1.26e5	D3	0
C21	5037.4		

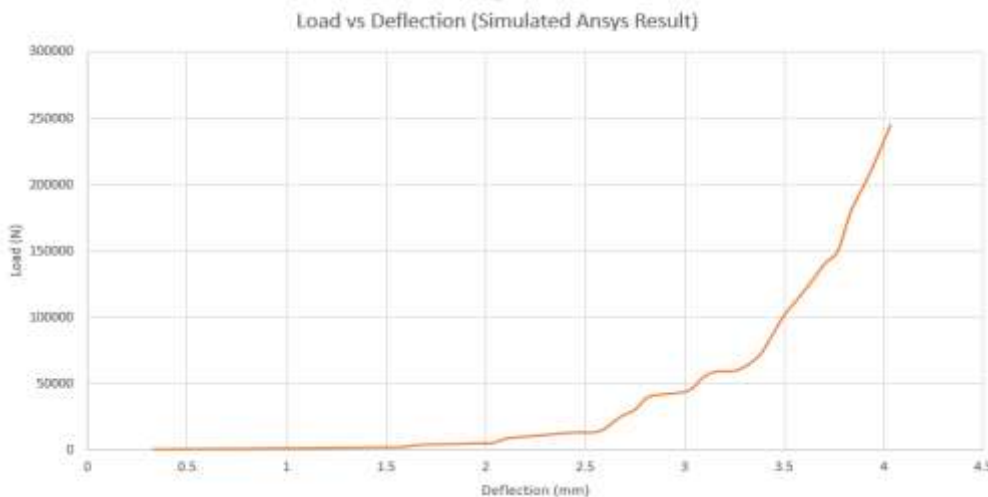


Fig.6 Load vs. Deflection ANSYS Results

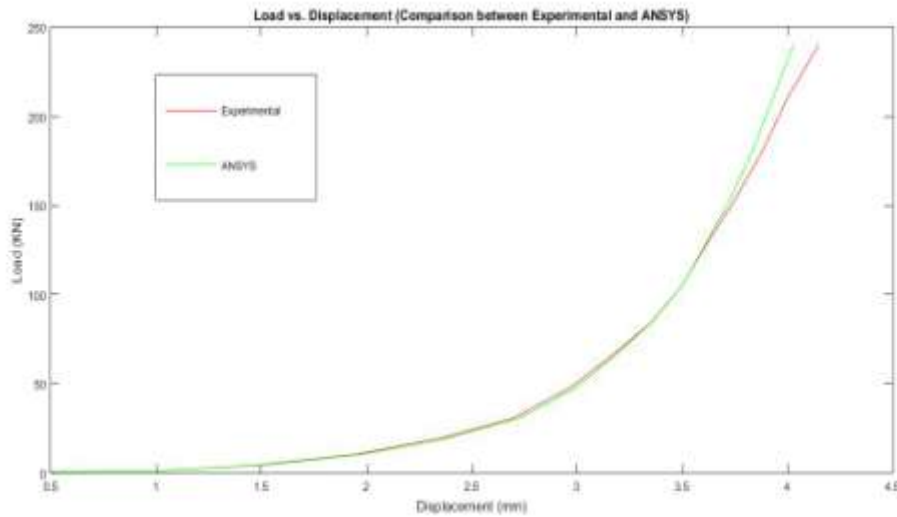


Fig.7 Load vs. Displacement comparison between ANSYS and Experimental Results

VI. Ansys Simulation

In further study, we have developed a beam model having cross-section as similar to 60kg Rail section, Length of beam taken as 36m so as to obtain variation over the long length. The rail pad under the rail considered as spring and stiffness of rail pad is given as input characteristics to the spring. Distance between two springs kept 0.6 m. The model developed will be as similar to the beam on elastic foundation. Considering point load at center; we have simulated the model for various load steps from (0-250 kN) and obtained plot of length vs. deflection from it.

RESULT TABLE:

Table 2: Linear and Non-Linear values of Rail-Pad

Load(R)	Linear Rail pad		Non-linear Rail pad	
	THEORY	ANSYS	THEORY	ANSYS
10	0.1031	0.1185	1.7293	1.7445
20	0.2062	0.1983	1.984	1.942
30	0.3093	0.3008	2.076	2.146
40	0.4124	0.4303	2.197	2.251
50	0.5155	0.5038	2.351	2.4632
60	0.6186	0.6571	2.793	2.7842
70	0.7217	0.7328	2.996	2.9809
80	0.8248	0.8122	3.013	3.1485
90	0.9279	0.9318	3.194	3.3167
100	1.013	1.05	3.225	3.35

Table 3: Theoretical and ANSYS Results for Maximum Displacement in Rail

	THEORETICAL	ANSYS
Linear	1.013	1.05
Non Linear	3.225	3.35

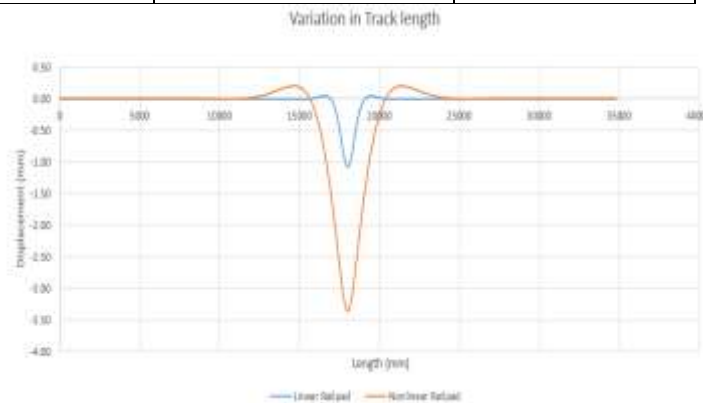


Fig. 8 Displacement Variation along the Length of Rail

VII. Inferences Drawn From The Results

A close examination of above results reveals the following observations:

1. The value of maximum deflection in rail varies by approximately three times in case of non-linear study as compare to linear study.
2. Experimental and simulated results validate the property of rubber pad whose stiffness varies with respect to load due to non-linear properties.
3. Deflection of rail along the length is different in case of linear simulation as compare to non-linear one.
4. Non-linearity of rail pad affects the behaviour of rail under loading.
5. Distance between zero deflection points and the load points is more in case of non-linear study than linear one.

VIII. Conclusions

1. In this paper, non-linear behaviour of rail pad has been investigated. Non-linear rail pad model developed using Polynomial model.
2. An exponential expression is derived for the non-linear stiffness of rail-pad model using curve fitting procedure using MATLAB.
3. The material modelling for rail pad has been developed to obtain the results of non-linear behaviour in ANSYS.
4. Experimental results have been validated by performing simulation in ANSYS.

Conflict of interest the authors declare that there is no conflict of interests regarding the publication of this paper.

References

Journal Papers:

- [1]. Samuel G. Coroma, Mohommad FM Hussein and John S. Owen, Influence of Preload and Non-Linearity of Rail pads on Vibration of Railway Tracks under Stationary and Moving Harmonic Loads, *Journal of Low Frequency Noise, Vibration and Active Control*, Vol.34 No.3 2015.
- [2]. Shane Crawford, Martin Murray, John Powell, Development of Mechanistic Model for the Determination of Track Modulus, 7th *International Heavy Haul, Brisbane, Australia*, 2001.
- [3]. K. Ganesh Babu, C.Sujatha, Track Modulus Analysis of Railway Track System using FEM, *Journal of Vibration and Control*, DOI: 10.1177/107754639341600, 2009.

Theses:

- [4]. Leposava Puzavac, Zdenka Popovic, Lukarazevic, *Influence of Track Stiffness on Track Behaviour under vertical Load*, University of Belgrade, Serbia, Promet-Traffic and Transportation, Vol.24, 2012, No.5, 405-412, 2012.
- [5]. Sakdirat Kaewunruen, Alexander Remennikov, *State Dependent Properties of Rail Pad*, University of Wollongong, Australia, 2009.
- [6]. S. Parvanova *Beams on Elastic Foundation*, University of Architecture, Civil Engineering and Geodesy, 2011.
- [7]. Joahannes Jacobus Heunis, *Material Models for Rail pad*, Stellenbosh University, March 2011.
- [8]. Minglei Ju, Handi Jmal, Raphael Dupuis, Evelyne Aubry, A Comparison among Polynomial model, Reduced Polynomial model and Ogden model for polyurethane foam, Laboratoire MIPS, university de Haute Alsace, 68093 Mulhouse France.

Chapters in Books:

- [9]. Prof. S K. Maiti, *Lecture 39, Advance strength of Material*, (Department of Mechanical engineering, IIT Mumbai.)
- [10]. Chapter 4 Elastic Foundations- <https://www.mae.ust.hk>.
- [11]. <http://www.rail-fastener.com/functions-of-rubber-rail-pads.html>